JOURNAL OF COMPUTATIONAL PHYSICS **126,** 237–239 (1996)
ARTICLE NO. 0132

MINIREVIEW

A Brief Perspective on Computational Electromagnetics

ARJE NACHMAN

Air Force Office of Scientific Research, AFOSR/NM, 110 Duncan Avenue, Bolling AFB, DC 20332

There is a growing interest in many quarters in acquiring the ability to predict all manner of electromagnetic (EM) effects. These effects include radar scattering attributes of objects (airplanes, missiles, tanks, ships, etc.); the mutual interference of a multitude of antennas on board a single aircraft or ship; the performance of integrated circuits (IC); the propagation of waves (radio and radar) over long distances with the help or hindrance of complicated tomography and ionospheric/atmospheric ducting; and the propagation of pulses through dispersive media (soil, treetops, or concrete) to detect pollutants or hidden targets, or to assess the health of runways. All of the above require extensive computation and, despite the fact that Maxwell's equations are linear in all these cases, codes do not exist which will do the job in a timely and error-controlled manner.

The radar scattering prediction problem is mainly one of computational size. If the radar is a standard monofrequency (or in reality narrow-band) unit, Fourier transforming the time derivative in Maxwell's equations to produce a frequency-domain system (in the scalar acoustics parlance one would have a Helmholtz equation) is desirable. The next good idea is to reconfigure the elliptic partial differential equation (PDE) system into an integral equation (IE) system using a free-space Green's function (kernel) and to integrate over the scattering object's surface. This not only reduces the problem dimensions by 1 but also eliminates the necessity to enforce numerically the Sommerfeld radiation condition. Discretizing this integral equation for the surface currents yields a full, dense matrix (the impedance matrix) that must be inverted to obtain the unknown currents. If there are N such unknowns, direct inversion requires $O(N^3)$ operations, as well as $O(N^2)$ storage. To maintain accuracy, N must grow as the square of the frequency, so that three-dimensional scatterers of even moderate size exhaust the capacities of today's computers, and increasing the linear dimensions of the scatterer by a factor of 10 will require 10^6 times more operations and 10^4 times more storage. Current research based on the special properties of the impedance matrix has lowered the number of operations to $O(N^{3/2})$ for large problems, with the promise of order N methods on the horizon. Such "fast multipole methods" (FMM) have been pursued by several groups since the method was introduced by Rokhlin [1]. Five years ago the largest nontrivial object whose scattering could be predicted was literally no larger than a breadbox. Since an airplane is hundreds of wavelengths long (the proper way to measure an object whose scattering details you wish to predict in the frequency domain), it is clear why serious computational electromagnetic (CEM) research in the frequency domain needed to be done. The numerical analysis mentioned above have, fortunately, also paid attention to error analysis and control [2].

As strong as the FMM/IE approach seems to be, it still has shortcomings. It does not handle well the long air intakes (ducts) for jet engines for example. Nor does it handle the part of the impinging EM field which penetrates the fuselage (through seams, etc.), the cockpit, and the radome and subsequently scatters off the airplane's own radar unit or the engine and its rotating compressor blades. Such effects do need to be accounted for since they are real and comprise a sizable part of the signature. Work on finite element (FEM) versions of the frequency-domain PDEs is being pursued to address these penetrable media (volumetric) problems. A good description can be found in [3]. Error control is only now being addressed for this approach, and it, along with every other approach which discretizes the region between the scatterer and some artificial/numerical boundary, must grapple with the "absorbing boundary condition" (ABC) which plays the role of the Sommerfeld radiation condition. Since ABCs are local, the resulting scheme is as fast as a standard finite element computation. The price is that the ABC may introduce errors in the calculation. Thus either the absorbing boundary must be sufficiently far from the scatterer in order to get accuracy, or something suitably clever must be done as regards near/on surface ABCs [4]. It is also difficult to judge the error introduced by the ABCs. One possible way to avoid the ABC vexation is to make a hybrid FE/IE scheme wherein the IEs are used on the artificial

0021-9991/96 $18.00

boundary to account accurately for the infinite domain. There are two interesting areas of theoretical research here. The first is the analysis of the conditioning of the "capacitance matrix" coupling the IE and FE solutions. It should be possible to use an iterative method to decouple the FE and IE solutions. Once this is done, the IE equations could be solved by a fast technique (i.e., FMM) independently of the FE solution, making the IE part of the code negligible in terms of time. The second area of research is the analysis of the convergence of this coupled system.

Since actual-sized scatterers are still unsolved computationally, one might ask what is done now to predict the signature of large objects. The answer is that needs are currently met using codes based on the high-frequency approximations to Maxwell's equations. Research is being pursued to upgrade the fidelity of these codes. Since the computer-aided design (CAD) files of targets are still for the most part flat-faceted, and since these ray-tracing-like codes require surface normals to make their predictions, the correct determination of normals (particularly at vertices and edges) by differential geometry [5] is regarded as an important contribution. Differential geometry is also playing an important role in the antenna interference problem. Instead of representing an airplane as a cylinder with flat plates attached (wings and tail!), it is now possible to predict EM coupling paths (geodesics) from actual CAD files of the airplane. Finally, these codes suffer from two common high-frequency illnesses, namely "erroneous shadow boundaries" and poor prediction of scattering in directions other than directly back. This latter impacts multistatic radar detection concerns. A cure seems possible by means of the addition of incremental length diffraction coefficients (ILDC) in a new, computationally efficient version [6]. The ILDCs represent a method of computing the contribution of edge currents. This is achieved by dividing diffracting edges into small segments and summing the contribution from all segments. Finally, I should mention work by Fatemi *et al.* [7], wherein automatic generation of multivalued solutions of the eikonal equation is accomplished together with superior shadow boundary descriptions.

Researchers in the long-distance wave propagation problem attempt to exercise the parabolic approximation to the wave equation. Recent interesting work involving Pade approximants has been reported [8]. Since safer attack routes may (or may not!) involve flying near hilly terrain, the scattering of these millions-of-wavelengths regions must be predictable.

One of the most interesting and potentially powerful CEM tools is the "method of boundary variations," which is basically a perturbation/series expansion approach [9]. In simple terms the method says that if one knows the solution to the scattering from a sphere, then one can deduce the scattering from a golf ball, with the perturbation parameter being the amplitude of the indentations. This ability to vary a boundary while exploiting the solution to some canonical shape is very important when one considers how many ways an F15 fighter aircraft can appear when loaded with all its different stores (fuel tanks, missiles, bombs, etc.) or how a tank can trundle along not only with its hatch open but also with various dents caused by recent engagements. The use of pertubation methods in EM scattering computations goes back to Rayleigh and his studies of diffraction gratings. The numerical potential of perturbative techniques has not been exploited until recently, however, due, in part, to a rather generalized perception of such methods as being unsound. A new understanding of the analytic structure inherent in perturbation expansions and the use of techniques that can produce appropriate analytic continuations has led to rigorous and efficient perturbative algorithms [9]. In fact, in many two- and three-dimensional applications, such algorithms have produced some of the most accurate results available.

Let us now turn to some time-domain work. Some of this work is driven by the interest in designing and operating pulsed (wide band) radars or other pulsed sources (high-power microwaves), and some of it is driven by the novel responses or returns that irradiated objects produce when pulsed. This latter subject embraces the broad concerns of wave propagation through dispersive media. Can we identify, for example, the radar return (from a pulsed source) that has been reflected from a tank hiding beneath a tree, or from a pool of pollutants beneath the ground? Since dispersion is irrelevant for monofrequency sources, why not confine ourselves to those and avoid unnecessary complications? Part of the reason for using pulsed sources lies in the fact that these target identification problems are contaminated by clutter, and it is hoped that some particularly discernible features in the dispersive case might be identified. In the case of pollutants there is the further possibility of "shaping" the incident pulse so that the EM precursor which is ultimately formed [10] will perform *in situ* remediation (microwave cooking). There is also the hope that suitably focused beams can be directed at cancerous tumors. Most human cells are known to open pathways (pores) when exposed to EM pulses and would thus be predisposed to ingesting chemotherapeutics which unexposed neighboring cells would ignore. Clearly the design of a source that produces such "suitably focused beams" is the real challenge here!

Before turning to the time-domain numerics, I wish to emphasize that the notion that time-domain problems can be solved by inverse Fourier transforming frequency-domain solutions is counterproductive. Not only would one have to have acquired the solutions for very many frequencies, but also the solution, if it were so obtainable, would most likely not be in the most suitable, edifying form. The CEM community has the challenge of devising codes for hyperbolic equations that do not suffer from any of several maladies. One such malady is discretization-induced dispersion. Work by Petropoulos [11] and others has clarified this phenomenon in standard finite-difference time-domain (FDTD) codes. However, clarification is not a cure. Raising the order from second (as is the current case) to something higher is not without its downside. As usual for higher order FD schemes, the boundary conditions are a problem, although work by Nicolaides has suggested a different higher-order generalization that looks promising. What to do when the medium really is dispersive has also been studied. In this case Petropoulos has shown [12] that a hierarchy of waves can be identified and their speeds, etc., depend on the particulars of the polarization dynamics. Another FDTD malady comes from the gridding. At present a uniform grid is used, but this has problems near curved surfaces. Recourse to a distorted grid near the curved boundary has controversial effects on the error, while overlapping grids are rumored to have long-time stability problems. A covolume unstructured FDTD scheme has been proposed, but its main problem seems to be the need for a Delaunay–Voronoi grid with ''well-behaved'' tetrahedra. The time-domain schemes also requrie ABCs. Recently a ''perfectly matched layer'' has been described by Berenger [13]; even more recently, work by Grote and Keller [14] has been published wherein the artificial boundary is a sphere and the untruncated condition on the sphere is exact so that no spurious reflections would be produced. The truncation which any real numerical implementation necessitates is also described and looks very impressive.

The finite-volume time-domain schemes have been based in computational fluid dynamics (CFD) methodology (see [15] and [16]) and use characteristics together with unstructured grids. While these are currently low-order schemes, they are very attractive when it comes to complex geometries. Much the same can be said for finite-element time-domain codes. Possibly the introduction of higher-order elements (h-p methods) needs looking into, as does the degree to which grid anisotropy and nonuniformity affect the solution of Maxwell's equations. It is possible that CFD grid generators do not produce optimal grids from the point of view of CEM. The related question of whether it is possible to develop an adaptive time-domain solver is also interesting and worth exploring.

I have not mentioned other approaches such as spectral methods because little in the way of published work is available, although interest in exploiting this method has been expressed. I will conclude by repeating that rigorous error control is crucial here just as it is in other computational endeavors.

ACKNOWLEDGMENT

I thank Drs. Monk, Petropoulos, Shankar, Fatemi, and Bruno for their valuable help.

REFERENCES

1. V. Rokhlin, *J. Comput. Phys.* **86,** 414 (1986).
2. W. C. Chew and C. C. Lu, *IEEE Trans. Antennas Propag.* **43,** 1483 (1995).
3. J. L. Volakis and L. C. Kempel, *IEEE Comput. Sci. Eng.* **2,** 42 (1995).
4. F. Harfoush, A. Taflove, and G. Kriegsmann, *J. Comput. Phys.* **89,** 80 (1990).
5. V. Oliker, private communication.
6. P. M. Johanssen, ''PTD Equivalent Edge Currents for Truncated Wedges,'' Rome Lab report RL-TR-95-217 (Sept. '95).
7. E. Fatemi, B. Engquist, S. Osher, *J. Comput. Phys.* **120,** 145 (1995).
8. R. Brent and J. Ormsby, *J. Phys. A Math. Gen.* **28,** 2065 (1995).
9. O. Bruno and F. Reitich, *J. Opt. Soc. Am. A Opt. Image Sci.* **10,** 1168 (1993); see also *J. Opt. Soc. Am. A Opt. Image Sci.* **10,** 2307 (1993).
10. K. Oughstun and G. Sherman, *J. Opt. Soc. Am. A Opt. Image Sci.* **6,** 1394 (1989).
11. P. Petropoulos, *IEEE Trans. Antennas Propag.* **42,** 62 (1994); see also *IEEE Trans. Antennas Propag.* **42,** 859 (1994).
12. P. Petropoulos, *Wave Motion* **21,** 253 (1995).
13. J. P. Berenger, *J. Comput. Phys.* **114,** 185 (1994).
14. M. J. Grote and J. B. Keller, *J. Math. Phys.* **122,** 231 (1995).
15. J. S. Shang, *IEEE Antennas Propag. Magazine* **37,** 15 (1995).
16. V. Shankar *et al., Electromagnetics* **10,** 127 (1990).